1 Introduction

Inertial navigation is an autonomous process of computing position location by doubly integrating the acceleration of a point whose position is to be determined. The fundamental concept is illustrated in Figure 1-1.

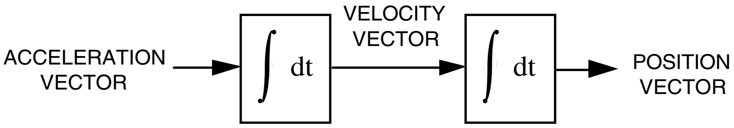

Figure 1-1 Fundamental Inertial Navigation Concept

Figure 1-1 shows a three-dimensional acceleration vector being integrated once to determine a three-dimensional velocity vector and again to obtain a three-dimensional position vector. Also implicitly represented in Figure 1-1 is the requirement to initialize the velocity/position integrators prior to the start of inertial navigation. In general, the initialization process requires knowledge of starting velocity and position.

An inertial navigation system (INS) implements the Figure 1-1 concept using a cluster of accelerometers to sense the acceleration vector components and a digital computer to perform the integration operations. The direction of the accelerometers (and the associated acceleration vector) is determined using a cluster of angle or angular rotation sensing instruments (e.g., gyros) that are physically mounted in a known geometrical relationship relative to the accelerometers. To ideally implement Figure 1-1 in the INS, the accelerometers would be specified to provide measurements of total acceleration (i.e., the second derivative of position). In general, total acceleration is composed of two fundamental parts: gravity acceleration created by the gravity field surrounding the INS, and “specific force” acceleration produced by forces acting on the vehicle containing the INS (which through mechanical linkage, produce forces within the INS accelerometers). Due to basic limitations of fundamental physics, accelerometers can only be designed to measure the specific force component of acceleration. Hence, to determine total acceleration for Figure 1-1, the gravity acceleration must be added to the accelerometer measurements. The result is the fundamental inertial navigation system concept depicted in Figure 1-2.

Figure 1-2 shows the gravity acceleration being calculated as a function of INS computed position. The gravity calculation is performed within the INS computer. Implied by this operation is a computerized model of the gravity vector field as a function of position in the space in which the INS is to be operated. The INS is mounted within a "user vehicle" whose position is to be calculated by the INS. Thus, by calculating its own position, the INS also determines the position of the vehicle in which it is mounted. Two classical INS mechanization approaches have been utilized to generate the specific force acceleration vector from the accelerometers (i.e., vector components and direction): the "gimbaled" approach and the "strapdown" approach.

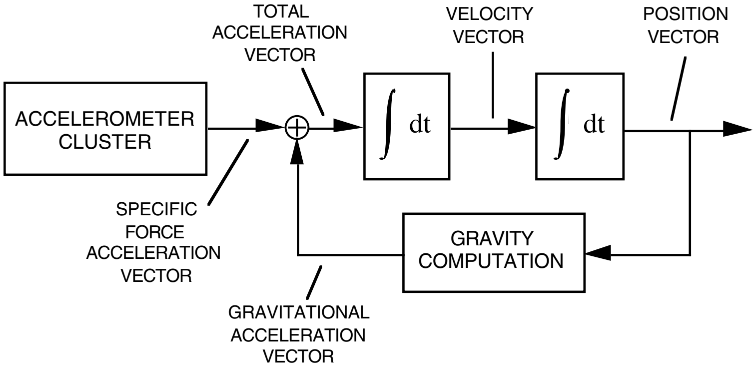

Figure 1-2 Fundamental Inertial Navigation System Concept

In the gimbaled approach, the accelerometers are mounted to a rigid structure that is mechanically coupled to the user vehicle by a set of concentric gimbals. The gimbals are connected to the accelerometer mount, to each other, and to the user vehicle by bearing assemblies that provide rotational freedom around the bearing axes. The "gimbaled platform" concept is depicted in Figure 1-3.

In Figure 1-3, three accelerometers (the cube structures) are mounted to the inner platform with their input sensing axes orthogonal. The inner platform angular orientation is controlled by electric torque motors mounted around the gimbal bearing axes. The control signals for the gimbal torque motors are provided by inertial angular rotation sensing instruments (gyros) mounted to the inner platform (cylindrical structures) with input sensing axes (dashed lines) orthogonal. By controlling the gyro outputs to be zero through the resulting gimbal torque motor closed-loop servo action, the inner platform (with its accelerometers) is controlled to maintain a specified angular orientation. To make the platform rotate at a prescribed angular rate (selected by the INS computer), the platform gyros are electrically biased by computer specified platform rotation rates, using biasing elements contained within each gyro. The platform gyro outputs are proportional to the integrated difference between the angular rotation rate dynamically input to the gyros (about their input axes) and the electrically applied gyro bias inputs. The gimbal torque motor control loops maintain the gyro outputs at zero, hence, the dynamic angular rate into the gyros (i.e., the inner platform angular rate) is forced to balance the gyro electrical biasing rate. The net effect is that the inner platform (and the accelerometers) are controlled to match the INS computer software specified integrated angular rate orientation profile (known in the INS computer), hence, the angular orientation of the accelerometers becomes implicitly known in the INS computer. The computer is then able to define the specific force vector using the accelerometer outputs for the vector component values, and the known orientation of the inner platform (the "inertial sensor platform") for the vector direction.

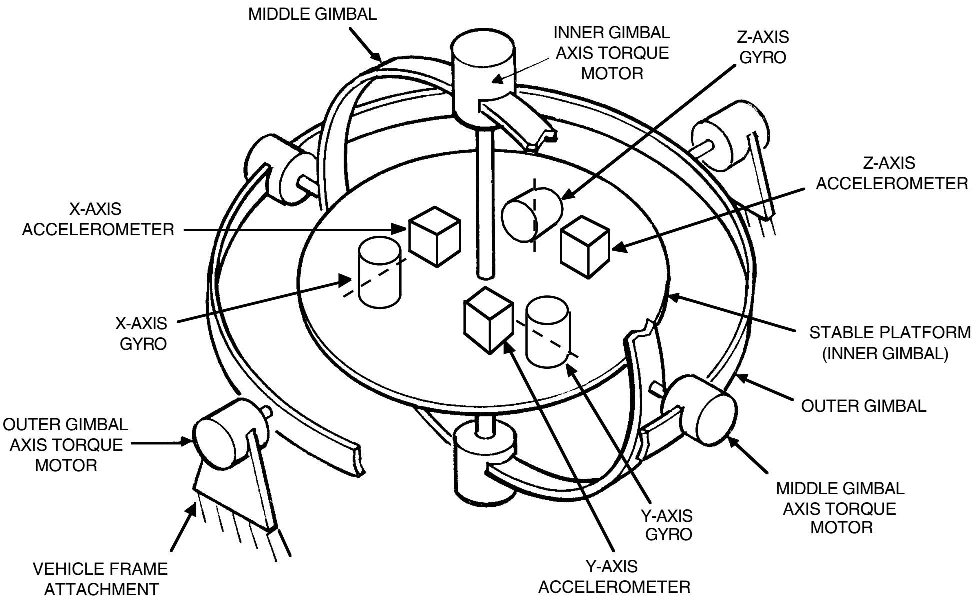

Figure 1-3 Gimbaled Inertial Platform Concept

A complete gimbaled INS consists of the Figure 1-3 "gimbaled platform", the INS computer, and associated electronics, all contained within a common chassis. The INS chassis is then physically connected to the user vehicle using a rigid "INS mount" assembly. Figure 1-4 illustrates the gimbaled platform interfaced to a computer in a gimbaled inertial navigation system. The navigation computation block in Figure 1-4 performs the Figure 1-2 integration and gravity computation operations.

In the strapdown approach, the interconnecting gimbal structure of Figure 1-3 is eliminated, and the inertial sensor platform (containing the inertial sensors) is mounted directly within the INS chassis (i.e., "strapped down" to the INS and to the user vehicle, thus the name "strapdown" to describe the technology). To perform the accelerometer orientation determination function (provided mechanically by the gimbal assembly in Figure 1-3), the

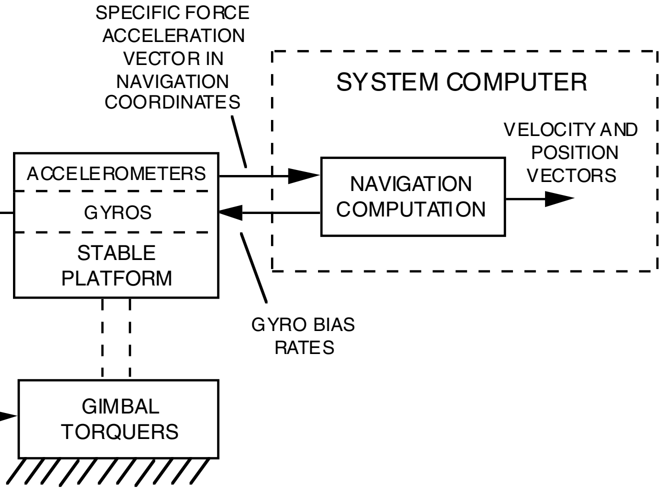

Figure 1-4 Gimbaled Inertial Navigation System

strapdown INS calculates the orientation of the strapdown accelerometers by processing a sensor assembly angular rate vector measured by the strapdown angular rate sensors (i.e., the so-called “body rate” signals) in the INS computer. (Henceforth, we will use the term “angular rate sensor” to generically define inertial sensors that sense angular rate. The more commonly used “gyro” term refers to inertial sensing instruments based on gyroscopic rotating mass dynamic principles. Modern day strapdown inertial sensors are based primarily on optical or Coriolis vibrating mass principles, hence, technically should not be called gyros, even though they measure the same input sensed by classical spinning mass gyroscopes. Spinning mass gyroscopes are also angular rate sensors, either directly, or in an integral sense). Figure 1-5 illustrates the strapdown INS for comparison with the Figure 1-4 gimbaled INS.

Both the strapdown and gimbaled system concepts in Figures 1-4 and 1-5 provide the same specific force acceleration vector inputs to the velocity/position integration navigation computation software in the system computer (i.e., the Figure 1-2 specific force acceleration vector). In the gimbaled system, the specific force vector is measured directly by the accelerometers on the gimbaled platform whose orientation (in the form of a “navigation coordinate frame” attitude) is selected (and controlled) by the navigation computer software. The resulting specific force vector in navigation coordinates is then processed as in Figure 1-2 to determine velocity and position. In the strapdown system, the specific force acceleration is first measured by the strapdown accelerometers as a vector in a “strapdown sensor coordinate frame”, and is then analytically rotated (by the INS computer software) from the strapdown sensor coordinate frame into the navigation coordinate frame. The result is the specific force vector in navigation coordinates used in Figure 1-2 for integration into velocity/position. To perform the coordinate frame rotation operation (called a “vector transformation”), the angular orientation between the strapdown sensor and navigation coordinate frames must be known in the system computer. It is found by a software integration operation using sensor coordinate frame angular rates measured by the strapdown angular rate sensors, and navigation coordinate frame angular rates specified by the INS software. The navigation coordinate frame angular rates are the same signals used in Figure 1-4 to bias the angular rate sensors in the gimbaled platform. Thus, both the strapdown and gimbaled systems generate the same navigation coordinate frame version of the specific force vector (for the Figure 1-2 input) and both use the same navigation frame angular rates in finding (or controlling) the specific force vector component coordinate frame.

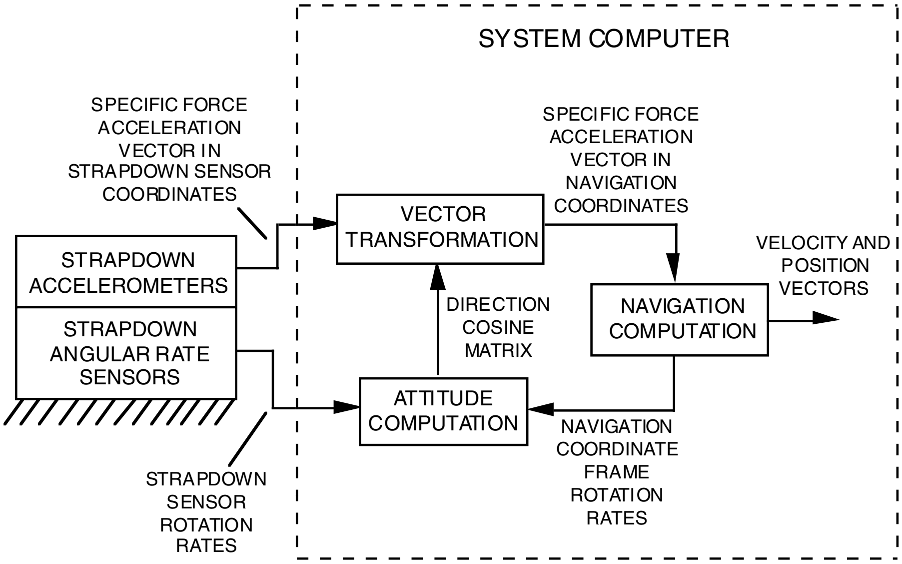

Figure 1-5 Strapdown Inertial Navigation System

In a general sense, the difference between a strapdown and a gimbaled system can be considered as a tradeoff between mechanical complexity (for the gimbaled system) versus computational complexity (for the strapdown system). From a performance standpoint, a fundamental handicap for the strapdown system is that the strapdown sensors (particularly the angular rate sensors) are exposed to the full vehicle angular rotation rate, whereas for the gimbaled system, the inertial sensor platform rate is controlled to be small, independent of vehicle angular rate. Meeting specified angular rate sensor accuracy requirements under high dynamic vehicle angular rate inputs (i.e., for the strapdown system) is generally more difficult to achieve than for the low benign angular rate environment of the sensors in a gimbaled platform. In fact the basic gimbaled platform concept was originated as a means of shielding spinning wheel gyros from vehicle angular rates, thereby making it possible to design gyros that would meet system accuracy requirements. With the advent of the ring laser gyro in the mid 1970's (an angular rate sensor based on optical rather than spinning mass dynamic principles), it became possible to achieve high accuracy under high angular rates. During the same time period, advancements in computer technology made it possible to implement the added strapdown computational burden for virtually no production cost penalty. The merging of these two technologies initiated the conversion of inertial navigation from original gimbaled to modern day strapdown technology. With few exceptions, the conversion process was complete by the year 2000. Today, strapdown inertial navigation and inertial navigation are synonymous.

The technologies utilized in the design of modern strapdown inertial navigation systems include inertial sensors (angular rate sensors and accelerometers), electronics (digital and analog), software, mechanical and thermal design, testing, and associated analytics. This book, in two parts, deals only with the analytical aspects of strapdown inertial navigation for the software resident in the navigation computer, software for system testing, and system performance analysis. For information on strapdown inertial sensors, the reader is referred to available literature on the subject (e.g., References 16, 21, 31 and 32), and is encouraged to contact the manufacturers of particular inertial components for further detail. Similarly, the reader should contact the manufacturers of inertial navigation systems for particulars on currently available system technologies. The remainder of this Chapter 1 Introduction describes the analytical material covered in each chapter of the book.

Chapter 2 provides a comprehensive guide to the terminology used throughout the book including mathematical notation, coordinate frame definitions and parameter definitions. Due to the diversity of analytical topics covered, it became virtually impossible to adopt a single meaning for each parameter and coordinate frame used throughout the book. To circumvent this difficulty, a Parameter Index and Coordinate Frame index is provided in the back of each of the Part 1 and Part 2 book volumes (in addition to the Subject Index) to facilitate locating parameter/coordinate frame definitions in the main text. The parameters and coordinate frames are alphabetically listed in these indexes with the equation number preceding their definition in the main text. In addition, a listing of mathematical symbols used is provided in Section 2.1 of Chapter 2, also with the equation number preceding their definition in the main text. To facilitate the recall of parameter, coordinate frame, and mathematical symbol definitions, they are separated and indented from each paragraph throughout the book, and repeated in sections that are far separated from sections in which they were first defined. The overall intent is to avoid the problem readers have found with many textbooks of forgetting the meaning of a particular variable and having to spend frustrating time trying to find its definition buried in the main text.

Chapter 3 provides an introduction to the basic mathematics utilized throughout the book including vector operations in selected coordinate frames, their analytical conversion process between coordinate frames, their component rates of change in rotating coordinates, and basic analytical operations for describing coordinate frame angular orientation ("attitude"). Attitude parameters discussed are the direction cosine matrix, the rotation vector, Euler angles, and the attitude reference quaternion, including analytical equivalencies between the parameters, and the analytics used to describe their rates of change. The concluding section provides a detailed discussion on methods for describing attitude and vector error characteristics. For the attitude error discussion, error angle vectors are derived describing the orientation error in the relative attitude between two coordinate frames, considering one of the frames as the reference and the other as having the orientation error. For the velocity error discussion, velocity error equations are developed as a function of the coordinate frame in which the velocity error is defined and the coordinate frame in which the error is to be evaluated.

Chapter 4 uses the Chapter 3 analytics to develop the equations that would typically be implemented in an earth based strapdown inertial navigation system computer for calculating attitude, velocity and position (as in Figure 1-5). The attitude/velocity/position calculations are analytically described in the form of time rate differential equations that would be continuously integrated in the INS computer using suitable digital integration algorithms. For the attitude determination function, both direction cosine matrix and quaternion forms are presented. Velocity is defined as INS position rate relative to the earth. The velocity vector rate equation is developed for integration in a locally level "navigation coordinate frame" (e.g., of the azimuth wander or free azimuth type, both of which are described), and includes the effect of navigation coordinate frame and earth's angular rotation rate relative to non-rotating inertial space. The attitude rate equations are derived to relate the strapdown sensor coordinate frame to the locally level navigation frame. Strapdown acceleration transformation operations are included for converting the accelerometer measured specific force acceleration into its navigation frame equivalent, the specific force input to the velocity rate equation. The position rate equation is defined in two parts: altitude rate and the rate of change of a direction cosine matrix relating navigation coordinates to a specified earth fixed coordinate frame ("position direction cosine matrix"). Included in the altitude rate equation is a method for controlling vertical error build-up in velocity and position ("vertical channel divergence") using an input pressure altitude signal. Equations are developed for converting the computed attitude/velocity/position data to equivalent output formats (e.g., roll/pitch/heading Euler angle attitude, north/east/vertical velocity components, latitude/longitude/altitude position or a position vector from a selected earth fixed position location to the INS). Equivalency equations are also provided for converting one form of position representation to another. Chapter 4 includes a brief discussion on initialization requirements covered in detail in Chapter 6. At the conclusion of Chapter 4, a summary table is provided listing the principal Chapter 4 equations and the inputs required from other sections of the book for earth related parameters and initialization operations.

As part of the inertial navigation software, analytical models must be included describing gravity in the space potentially occupied by the INS and to describe the referencing surface for position definition (e.g., the surface of the earth including its rotation rate relative to inertial space). Chapter 5 analytically describes the earth in terms of its classically represented ellipsoidal reference surface (approximately at mean sea level), and the analytical definition of earth referenced parameters used in the INS computer (e.g., latitude, longitude, altitude above the earth reference ellipsoid, the relationship between INS horizontal velocity and the angular rate of the locally level navigation coordinate frame (called "transport rate"), and radii of curvature of the earth's surface used in calculating the transport rate). A section is included in Chapter 5 summarizing the classical Reference 3 and 4 gravity model used in most inertial navigation systems. A navigation coordinate frame version of the gravity model is developed in Chapter 5 for input to the Chapter 4 velocity rate equation (in a form known as "plumb-bob gravity" which includes a centripetal acceleration term associated with position relative to the rotating earth). In all cases, the equations are developed in complete closed-form without resorting to "first order approximations" prevalent in many navigation analytical documents. It is the author's belief that computer technology has now advanced sufficiently (in speed, word length, and higher order language utilization) that closed-form equations can be implemented without penalty in an INS computer. Significant advantages thereby accrue in algorithm performance and in accompanying software validation/documentation processes that need not address the accuracy of first order approximations. Chapter 5 concludes with a summary table listing the Chapter 5 equations that would be utilized in a typical INS computer.

In a strapdown INS there are three operations that must be initialized prior to engaging the inertial navigation "mode". These are the integration functions used to determine attitude, velocity and position. Chapter 6 addresses the analytics associated with performing the attitude/velocity/position integration function initialization operations in the INS computer for applications when the user vehicle is at a "quasi-stationary" attitude/position orientation (e.g., an aircraft on the ground with parking brake engaged, but under quasi-stationary attitude/position motion due to wind gusts, passenger/crew movement, fuel/stores loading). The attitude initialization process discussed utilizes a closed-loop "Kalman filter" aided integration process using inputs from the INS accelerometers and angular rate sensors to initialize the attitude orientation relative to the vertical and true north ("true heading"). The true heading initialization is achieved by estimating horizontal earth rotation rate components and using the result to initialize the heading attitude of the INS attitude parameters or the orientation of the navigation coordinate frame (i.e., the position direction cosine matrix). Chapter 6 also analytically describes a "Coarse Leveling" process by which an approximate vertical attitude initialization can be achieved using accelerometer inputs. Coarse Leveling is typically performed before engaging the previously described vertical/heading initialization process (known as "Fine Alignment").

Chapter 7 derives the equivalent digital integration algorithm form of the Chapter 4 differential equations for attitude/velocity/position determination in the strapdown INS computer. The attitude algorithm development section addresses both direction cosine matrix and quaternion forms for strapdown sensor attitude relative to the locally level navigation frame, each separately dealing with updating for strapdown sensor rotation (measured by the angular rate sensors) and for navigation frame rotation rates. The attitude algorithms are structured based on three repetition rates (per pass of the associated computation chain); a high speed rate for high frequency angular rate sensor inputs (e.g., angular vibration), a moderate speed attitude updating rate for angular rate sensor inputs (including summing of the high speed algorithm output), and a lower speed attitude updating rate for navigation frame rotation rates. Closed-form expressions (without approximation) are derived for all but the high speed algorithms. The high speed algorithm is derived as an approximation to an exact continuous form integral equation that measures what is known as "coning" effects in the attitude solution.

The velocity updating algorithms in Chapter 7 are also structured using a multiple speed architecture; a high rate algorithm to measure high frequency effects and a moderate speed algorithm to handle the velocity updating operation (including summing of the high speed algorithm output). Closed-form expressions (without approximation) are derived for all but the high speed algorithm. The high speed algorithm uses angular rate sensor and accelerometer inputs in an approximation of an exact continuous integral equation to measure what is known as "sculling" effects in the acceleration-transformation/velocity-updating operation. The moderate speed algorithm adds the sculling output from the high speed algorithm to summed increments of integrated accelerometer specific force output (including what is known as a "rotation compensation" correction), transforms the result to the navigation frame, adds plumb-bob gravity, and adds Coriolis acceleration effects (to account for navigation and earth rotation rate effects) to update the navigation frame velocity components.

Two forms of position updating algorithms are presented in Chapter 7; a classical set operated at a single repetition rate based on trapezoidal integration of velocity, and a "high resolution" set based on a multiple speed architecture similar to the attitude/velocity updating algorithms. For the high resolution approach, a high rate algorithm measures high frequency effects and a moderate speed algorithm handles the position updating operation (including summing of the high speed algorithm output). Closed-form expressions (without approximation) are derived for all but the high speed algorithm. The high speed algorithm uses angular rate sensor, accelerometer, and sculling algorithm inputs to measure what has been termed (by the author) "scrolling effects" in the position updating process. The lower speed algorithm adds the scrolling output from the high speed algorithm to computed increments of doubly integrated accelerometer specific force output (including a "position rotation compensation" correction - author coined name), transforms the result to the navigation frame, adds the position change due to velocity at the start of the update cycle, and uses the resulting navigation frame "position change increment" to update the position data (altitude and the position direction cosine matrix). The trapezoidal positioning algorithm computations are identical to the moderate speed portion of the high resolution algorithms, but with the position change increment calculated as a trapezoidal integration approximation for integrated navigation frame velocity.

For the Chapter 7 attitude, velocity and position multiple speed algorithms, the form of the algorithms is structured so that in situations when sufficient throughput exists (the trend for the future), the lower speed algorithms can be executed at the higher speed algorithm repetition rate to simplify the software executive control architecture. A table is provided at the conclusion of Chapter 7 listing the Chapter 7 and Chapter 5 computation algorithms that would be typically used in a high performance INS, in their order of execution in the INS computer.

A fundamental problem with all inertial navigation systems is the inability to manufacture inertial components with the inherent accuracy required to meet system requirements. To correct for this deficiency, compensation algorithms are included in the INS software for correcting the sensor outputs for known predictable error effects. Chapter 8 develops analytical equations for compensating the strapdown inertial sensor outputs. Inertial sensor compensation algorithms derive from classical analytical models used in the inertial sensor industry to characterize a sensor's output (including errors) as a function of the sensor input (error free). In contrast, the sensor compensation algorithms used in the INS computer are designed to translate the sensor outputs (containing error) to the equivalent error free form. Thus, the compensation algorithms represent the inverse of the inertial sensor analytical model equations. In many systems, the form of the compensation equations so derived contain linearization approximations to the exact inverse relations (to conserve on computer throughput). The approach taken in Chapter 8 is to use the complete inverse form (without approximation) based on the assumption that modern day computers of today (and certainly in the future) can handle the workload.

Chapter 8 is divided into four parts; development of the inertial sensor output compensation algorithms, developing algorithms for correcting the high speed portion of the attitude/velocity/position algorithms (i.e., coning, sculling, scrolling, integration of inertial sensor outputs between computation cycles) that may have been calculated using uncompensated inertial sensor data, compensation for misalignment of the strapdown sensor assembly relative to the INS mount installation in the user vehicle, and a summary section. The summary provides a tabulated listing of the compensation equations that might be used in a high performance INS, tabulated in the order of execution in the INS computer, and showing their application in conjunction with the Chapter 5 and 7 inertial navigation computation algorithms. Chapter 8 includes a discussion of methods for compensating quantization error on the strapdown inertial sensor signals. Also included is the derivation of algorithms for compensating the effect of physical displacement between the accelerometers in a strapdown sensor assembly (known as "size effect") which, under angular rotation, exposes each accelerometer to a slightly different acceleration vector. Intermediate computation results in the size effect algorithms are also applied for compensating anisoinertia angular rate sensitive error effects in pendulous accelerometers.

In some applications (e.g., Synthetic Aperture Radar), it is important that jitter motion of the strapdown inertial sensor assembly be removed from the computed INS attitude/velocity/position outputs. Chapter 9 provides a smoothing architecture for achieving such jitter compensation that avoids introducing dynamic distortion in the smoothed output signals.

Chapter 10 develops analytical techniques for evaluating the error in the high speed portion of the attitude/velocity/position algorithms under anticipated sinusoidal and random INS input vibrations. Strapdown inertial navigation integration algorithms are designed to accurately account for three-dimensional high frequency angular and linear vibration of the sensor assembly. Such motion, if not properly accounted for, can lead to systematic attitude/velocity/position error build-up. The high speed algorithms developed in Chapter 7 to measure these effects (i.e., coning, sculling and high resolution position updating routines) are based on approximations to the form of the angular-rate/specific-force profiles during the high speed update interval. An important part of the algorithm design is their accuracy evaluation under hypothesized vibration exposures of the strapdown INS in the user vehicle, the subject of

Chapter 10. Algorithm performance evaluation results, used in design/synthesis iterative fashion, eventually set the order of the algorithm selected and its required repetition rate in the INS computer.

Since the sensor assembly is dynamically coupled to the INS mount through the INS structure (in many cases including mechanical isolators and their imbalances), vibrations input to the INS mount become dynamically distorted as they translate into the resulting inertial sensor output vibrations provided to the navigation algorithms. Included in Chapter 10 is a review of linear dynamic system frequency response analytics and the development of a simplified analytical model for characterizing the dynamic response of an INS sensor assembly to input vibration. The sensor assembly dynamic response model is one of the elements utilized in the Chapter 10 algorithm performance evaluation equations presented. Chapter 10 includes an analysis of folding effect amplification in the position update algorithms induced by linear vibrations of the sensor assembly. Such effects are generally not present in the attitude/velocity algorithms because the inertial sensors are generally of the integrating type, providing their inputs to the navigation computer in the form of pre-integrated angular rate and acceleration increments. Chapter 10 also provides an analysis of coning/sculling algorithm error induced by inertial sensor dynamic mismatch. Chapter 10 concludes with an analytical description of a simple simulation program that can be used to evaluate high speed algorithm error under user specified INS sinusoidal and random vibration input exposure.

Chapter 11 deals with the validation of strapdown inertial navigation integration algorithms by computer simulation. It addresses the basic issue of how to analytically generate a "truth model" set of angular rate and specific force acceleration data representative of the output from ideal (error free) strapdown inertial sensors (typically in the form of integrated angular rate and specific force acceleration increments), and how to analytically generate a corresponding "truth model" attitude/velocity/position profile. Validation of the algorithms then consists of running the algorithms at their selected repetition rate(s) using the "truth model" sensor inputs, and comparing the algorithm attitude/velocity/position response to the equivalent "truth model" attitude/velocity/position profile. In general, two methods can be considered for the truth model; 1. A digital integration approach in which the truth model integration algorithms are more accurate than the INS algorithms being validated, and 2. Closed-form analytical equations representing the exact analytical integration of the angular-rate/specific-force profile. The problem with the Method 1 approach is the dilemma it presents in demonstrating the accuracy of a truth model that also contains digital integration algorithm error, allegedly smaller than the error in the INS digital integration algorithms being tested. Chapter 11 addresses the Method 2 approach, and derives closed-form analytically exact truth models for evaluating classical groupings of INS algorithms used to execute basic integration operations; 1. Attitude updating under dynamic conditions, 2. Attitude updating, acceleration transformation, velocity updating under dynamic conditions, 3. Attitude updating, acceleration transformation, velocity/position updating under dynamic conditions (including accelerometer size effect separation), 4. Attitude/velocity/position updating during long term navigation over an ellipsoidal earth model. Simulation programs for these functions are analytically described in Chapter 11 including the method of comparing the INS algorithm results with the truth model. A table is provided showing which algorithm equations (by Equation number used in the book) are validated with each simulator. Chapter 11 also includes a discussion of specialized simulators for use in validating particular algorithm portions.

The overall strapdown INS design process requires supporting analyses to develop and verify performance specifications for the INS components, particularly the inertial sensors. This generally entails the use of a strapdown INS error model in the form of time rate differential equations that describe the error response in INS computed attitude/velocity/position data. Such error models are also fundamental to the design of Kalman filters (the subject of Chapter 15) used, in conjunction with other system inputs, for correcting the INS errors. Chapter 12 provides a detailed description of the analytical process used in deriving strapdown INS error model equations that represent the INS integration routine response to sensor input errors (i.e., excluding the effect of algorithm and computer finite word-length error, errors that are generally negligible in a well designed modern day INS compared to sensor error effects). Chapter 12 is based on the error form of the Chapter 4 and 5 strapdown INS computation equations.

An important part of INS error model development is the definition (and selection) of attitude/velocity/position error parameters used in the error model and their relationship to the INS computed attitude/velocity/position parameters (or to a hypothetical set of INS navigation parameters that are analytically related to the INS computed set). Chapter 12 describes several versions of navigation error parameters that can be considered and the process followed in selecting one set for a particular application. After describing the general process used in developing INS error models, Chapter 12 derives error model equations for different error parameter sets using two approaches; 1. Direct derivation based on the error parameter definitions, 2. Derivation by conversion of a previously derived error model (based on one set of error parameters) to an alternative error model based on another error parameter set. The second method is based on equivalencies between error parameter sets derived in Chapter 12. Included in Chapter 12 is the analytical modeling of inertial sensor error inputs and modeling of error effects induced by sensor assembly vibration.

Chapter 13 deals with analytical solutions to the Chapter 12 strapdown INS error model equations under classical trajectory profiles and inertial sensor error characteristics. Such analyses are useful for understanding the nature of sensor error propagation into attitude/velocity/position under particular conditions, and for pencil-and-paper performance predictions. Chapter 13 begins with a general analytical description of INS error characteristics including vertical channel response (with gravity model induced exponential divergence/control), horizontal channel response (Schuler oscillations and long term “earth loop” effects), and the unique characteristics of strapdown inertial sensor scale-factor/misalignment error on INS navigation performance under dynamic angular rate conditions. Chapter 13 then develops closed-form solutions to the Chapter 12 equations under various simplifying assumptions for the sensor errors and trajectory profile (e.g., constant sensor errors under high rate spinning about stationary and rotating axes, in horizontal circular trajectories (in general and at Schuler frequency), and under long term cruise; random sensor error effects during short term and 2 hour trajectories).

Chapter 14 addresses the effect of strapdown inertial sensor error on the Chapter 6 quasi-stationary initial vertical/heading attitude initialization process. The error model for the Chapter 6 Fine Alignment initialization process equations is developed and solved in closed-form for constant and random output inertial sensor errors, ramping accelerometer error, inertial sensor quantization errors, and random vibrations. The random and quantization error analysis is based on solving the general continuous form Kalman filter covariance differential equation developed in Chapter 15.

The accuracy of all inertial navigation systems is fundamentally limited by instabilities in the inertial component error characteristics following calibration. Resulting residual inertial sensor errors produce INS navigation errors that are unacceptable in many applications. To overcome these deficiencies, "inertial aiding" is commonly utilized in which the INS navigation parameters (and in some cases, the sensor calibration coefficients) are updated based on measurements from an alternate source of navigation information available in the user vehicle (e.g., Global Position System (GPS) receiver provided data). The modern method for applying the inertial aiding measurement to the INS data is through a Kalman filter, a set of software that is typically resident in the INS computer. Chapter 15 describes Kalman filtering in general and how it relates to the aiding of strapdown inertial navigation systems. Included is a detailed introductory section that develops the basic theory of Kalman filter estimation in general, its interface/timing/synchronization architecture in the host computer, and procedures for software validation.

The Kalman filter theory developed in Chapter 15 is at the on-set, based on "optimally" estimating an "error state vector" representing the error characteristics of the device(s) providing inputs. This contrasts with classical Kalman filter theory based on estimating a "state vector" representing parameters in the input devices (e.g., position parameters in an INS). In inertial navigation applications, the error state vector selected for the Kalman filter is related to the computed navigation parameters (e.g., the three component attitude error vector described in Chapter 12 which is related to errors in the nine component attitude direction matrix or the four component attitude quaternion), but generally does not explicitly represent the errors in the computed parameters (as in the traditional "extended" Kalman filter approach). Developing the Kalman filter from scratch based on a general error state vector approach provides a direct method for arriving at the result used in most Kalman filter applications.

Chapter 15 develops discrete recursive forms of the Kalman filter (suitable for software implementation) and a general continuous form for performance analysis. Following the approach outlined in Reference 6, a general solution is developed for the continuous form Kalman filter. The result is then extended for the singular case of zero "measurement noise", a situation encountered in Kalman filters applied to the Chapter 6 inertial navigation system Fine Alignment process (and used in Chapter 14 to derive closed-form solutions for the Chapter 6 quasi-stationary initial alignment error equations). Chapter 15 provides examples of discrete form Kalman filter configurations applied to the Chapter 6 Fine Alignment process, to dynamic moving base INS initial alignment, to inertial aiding using a generic vehicle mounted velocity sensor measurement, and for inertial aiding using GPS position range measurements.

Inherent in the structure of Kalman filters is a statistical estimate of the uncertainties in the computed error state vector, typically represented in the form of an error state uncertainty "covariance matrix". The calculations involved in continuously computing the covariance matrix (it changes as a function of time based on the time profile of the navigation parameters, system/sensor error characteristics, and Kalman updating history) can also be used in a performance analysis time domain simulation program for statistically estimating the INS errors. Chapter 16 addresses the structure of such covariance simulation programs for application to INS performance assessment and as part of the Kalman filter design process. As a Kalman filter design aid, the covariance simulation is used to simulate the equivalent operations performed in the Kalman filter being designed/tested and, from a covariance standpoint, to evaluate the performance of the INS when aided by the Kalman filter. The aided INS performance analysis capability permits the user to account for all error effects being simulated (the so-called "real world" model) when interfaced with a Kalman filter based on an approximate version of the real world (the so-called "suboptimal" Kalman filter). The Kalman filter design process consists of using the simulation over a representative set of trajectory profiles, evaluating aided INS performance, and modifying the Kalman filter dynamic model (e.g., the number of error state vector elements or the magnitudes of included noise sources) in iterative fashion until performance satisfies user specified criteria. Included in the Chapter 16 covariance simulation program, is the ability to provide sensitivity outputs that identify the sensitivity of navigation errors to the error sources, and an "error budget" that shows the contribution of each error source to the navigation errors, both of which are useful during the Kalman filter design/iteration process.

Simulation analysis of strapdown inertial navigation systems often require the use of "trajectory generators", simulation programs that provide navigation parameter outputs as a function of time over a user selected trajectory profile. The Chapter 16 covariance simulation program requires such a trajectory generator input as does the process described in Chapter 15 for validating Kalman filters (and their internal computational elements). Chapter 17 deals with the design of trajectory generators that provide navigation parameter outputs as well as strapdown inertial sensor inputs in the form of integrated angular-rate/specific-force-acceleration increments (integrals between trajectory generator time points). The integrated inertial sensor increments are identical to the outputs from idealized strapdown inertial sensors (i.e., error free), with the trajectory generator navigation parameters then representing the output from an idealized error free strapdown inertial navigation system. Chapter 17 first describes the general requirements for a trajectory: 1. Trajectory shaping, an interactive process by which the user creates a trajectory profile to meet a general set of requirements, and 2. Trajectory regeneration in which the shaped trajectory is "played back" as part of a larger simulation program to regenerate the navigation/inertial sensor data history. Included must be the inherent characteristic of the navigation parameter outputs (attitude/velocity/position) and associated inertial sensor signals to be consistent with what would be obtained from an ideal integration of the inertial sensor data into trajectory navigation parameters.

The major portion of Chapter 17 provides a detailed description of a trajectory generator designed to produce realistic trajectories representative of maneuvering vehicles in the vicinity of the earth (i.e., aircraft, surface ships, underwater vehicles). Once the trajectory profile is created, the Chapter 17 trajectory generator provides (as options in the trajectory regeneration process) the ability to add aerodynamic angle-of-attack/sideslip effects, user vehicle structural bending effects, high frequency vibrations, and to simulate trajectories of different points in the same vehicle separated by flexible structure. The Chapter 17 trajectory regeneration function is structured as the analytical inverse of the Chapter 7 high accuracy strapdown inertial navigation algorithms (including high resolution position updating). This technique assures that integration of the trajectory generator inertial sensor signals with the Chapter 7 algorithms will produce the same navigation solution, the correct response under error free sensor and computer processing conditions.

Chapter 18 describes five system level tests that can be performed on a strapdown INS to ascertain the error characteristics of the strapdown inertial sensors; the Schuler Pump Test, Strapdown Drift Test, Repeated Alignment Test, Continuous Alignment Test and the Strapdown Rotation Test. Each can be executed in a test laboratory using a rotation fixture to which the INS is mounted. The Schuler Pump Test is based on amplifying the classic 84 minute sinusoidal Schuler error response characteristic of a strapdown INS (described in Chapter 12). Analysis of the velocity error response provides the ability to determine composite angular rate sensor and accelerometer errors that created it. The Strapdown Drift test is a static test in which the attitude integration software in the INS computer is configured to constrain the average horizontal transformed specific force acceleration to zero. For a test of several hours duration, the averages of the constraining signals become accurate measures of angular rate sensor bias error. The Repeated Alignment Test is a static test in which the Chapter 6 Fine Alignment process is repeated to generate a sample set of horizontal earth rate estimates at the end of alignment. By analyzing the variance in the end-of-alignment earth rate signals, the horizontal angular rate sensor random noise is estimated. The Continuous Alignment Test estimates horizontal angular rate sensor random noise using the time history of horizontal earth rate estimates taken during a single initial alignment run. The Strapdown Rotation Test consists of exposing the INS to a series of rotations, and recording its average transformed specific force acceleration output at static dwell times between rotations. By processing the recorded data, very accurate measurements can be made of the scale factor error and relative misalignment for all inertial sensors in the sensor assembly, the accelerometer bias errors, and misalignment of the sensor assembly relative to the INS mounting fixture. In each case, the test procedure is described and the analytics developed in detail for the associated data processing algorithms.

Chapter 19 provides three pertinent papers published by the author since the original publication of this book in 2000. The first paper derives from velocity/position algorithms developed in Chapter 7 that are designed to be exact under particular trajectory conditions (primarily, constant strapdown angular rate and specific force over the velocity update interval). Using the exact velocity/position updating algorithm structure as a base, high speed routines are derived for computing the algorithm input under general trajectory conditions. The result is a two speed velocity/position algorithm structure that directly parallels the two-speed attitude updating approach described in Chapter 7. The second paper provides an integrated and expanded treatment of material on sensor quantization error described in several sections of this book. Of particular interest are new sections rigorously describing how quantization error is properly modeled to account for different attitude/velocity/position algorithm update rates. The third paper addresses some fundamental questions on implicit assumptions used throughout the book regarding inertial sensor measurements. Gyros measure angular rate relative to non-rotating inertial space. Accelerometers measure specific force which when analytically combined with gravitational acceleration provide total acceleration for integration into velocity/position. Specific force has been defined as the acceleration relative to non-rotating inertial space produced by non-gravitational forces. But what exactly is non-rotating inertial space? What exactly is total acceleration? Is gravitation an absolute or a relative parameter? Can specific force be defined without reference to gravity? The third paper in Chapter 19 provides some interesting answers to these and other fundamental inertial sensing questions.

With the exception of this Chapter 1 Introduction, each chapter includes an introductory Overview section outlining the basic material to be covered. References for all chapters are provided in the back of each of the Part 1 and 2 book volumes.