

# Strapdown Analytics - Second Edition

**Paul G. Savage, STRAPDOWN ASSOCIATES, INC.**

This hard cover text in two volumes (Parts 1 and 2) is the second edition of *Strapdown Analytics*. It contains all material provided in the first edition including errata corrections uncovered since the original publication in 2000. This edition contains an additional Chapter 19 which presents three relevant strapdown papers published by the author since 2000.

Strapdown Analytics provides a detailed comprehensive discourse on the analytics of strapdown inertial navigation systems (INSs), the basic technology used on modern day commercial and military aircraft, guided missiles, surface ships and underwater vehicles. It develops the algorithms implemented in the strapdown system computer as well as providing the analytics associated with system software validation, system test, simulation, performance analysis, and the analytical design methodology used in deriving the strapdown equations. Included is an in-depth chapter dealing with Kalman filter theory and its application to the aiding of a strapdown INS.

The title of the book, *Strapdown Analytics*, has a double meaning. It deals with the analytics associated with strapdown inertial navigation systems; it also describes the analytics used at the author's company, Strapdown Associates, Inc. (SAI or "Strapdown" for short) over the 20 year period following its inception in 1980. During this time, SAI developed strapdown inertial navigation application software in both free-inertial and Kalman-filter-aided configurations for various private and governmental organizations including validation software, system test software, simulators, and associated performance analyses. In addition to

being SAI's president, one of the author's principal responsibilities was to prepare the technical documentation for these programs.

*Strapdown Analytics* has been prepared for the reader who may not have had experience in navigation or Kalman filtering. The analytical material presented is derived from scratch, showing the developmental steps in rigorous detail, without relying on reference material for supporting analytics. The book is complicated, yet complete and understandable by analytically inclined graduate students and practicing engineers. The book can be viewed as the text for an advanced course one might take following the introductory course taught by the author, *Introduction To Strapdown Inertial Navigation Systems*. To aid the reader, the book includes several unique features:

- Clearly delineated parameter definitions separated from the main text, including a parameter index for referencing to the book location where the parameter was defined.
- Repeating a parameter definition where needed for clarity in sections that are far removed from the section in which it was originally defined.
- Deriving equations beginning from basics; avoiding the practice of referencing supporting equations to other source material.
- Providing equation derivations that show the intermediate steps to avoid having the reader accept results on faith, or spend valuable time filling in the voids for verification.

## About The Author

Paul G. Savage is an internationally recognized expert in the design and test of ring laser gyro strapdown inertial navigation systems. He is the President and Chief Analyst of Strapdown Associates, Inc. (SAI), a Minnesota engineering company he founded in 1980 to further the advancement of strapdown inertial technology. Since the down-sizing of SAI in 1995, Mr. Savage has continued to provide inertial navigation consultation services and teach his *Introduction To Strapdown Inertial Navigation Systems* course to the aerospace industry.

From 1963 to 1980, Mr. Savage was employed at

Honeywell, Inc. as Senior Principal Engineering Fellow where he led engineering design teams in the evolutionary development of laser gyro strapdown inertial navigation systems. Mr. Savage was the engineering manager and system design engineer for the Honeywell LINS-0 strapdown inertial system, the first to prove the readiness of laser gyro strapdown inertial navigation technology for aircraft applications as demonstrated during a landmark flight test series at Holloman Air Force Base in 1975.

Mr. Savage is a graduate from the Massachusetts Institute of Technology where he received his MS and BS degrees in Aeronautical Engineering in 1960.

# Strapdown Analytics, Second Edition - Part 1

## Contents (841 Pages)

### 1. INTRODUCTION (16 Pages)

#### 2. TERMINOLOGY (8 Pages)

Mathematical Notation  
Coordinate Frame Definitions  
Parameter Definitions

### 3. VECTOR, ATTITUDE AND COORDINATE FRAME FUNDAMENTALS (96 Pages)

Vectors And Coordinate Frame Transformations  
Attitude Parameters (Direction Cosines, Quaternions, Rotation Vector, Euler Angles)  
Attitude Parameter Rate Equations  
Vector Rates Of Change In Rotating Coordinates  
Attitude And Vector Error Characteristics

### 4. CONTINUOUS FORM STRAPDOWN INERTIAL NAVIGATION EQUATIONS (43 Pages)

Attitude Rate Equations (Direction Cosine And Quaternion)  
Acceleration Transformation  
Velocity Rate Equation  
Position Determination  
Local Level Coordinate Frame Options (Wander Azimuth, Free Azimuth, Geographic)  
Initialization

### 5. EARTH RELATED NAVIGATION PARAMETERS (33 Pages)

Earth Shape Model  
Ellipsoidal Earth Referenced Navigation Parameters  
Transport Rate  
Gravity Model

### 6. QUASI-STATIONARY INITIALIZATION (19 Pages)

Attitude Initialization  
Navigation Frame Initialization  
Velocity Initialization  
Altitude Initialization

### 7. STRAPDOWN INERTIAL NAVIGATION DIGITAL INTEGRATION ALGORITHMS (95 Pages)

Attitude Update Algorithms (Direction Cosine and Quaternion, Coning )  
Acceleration Transformation, Sculling, And Velocity Update Algorithms

Position Update Algorithms (Conventional And High Resolution)  
Algorithm And Execution Rate Selection

### 8. NAVIGATION SYSTEM COMPONENT COMPENSATION ALGORITHMS (108 Pages)

Inertial Sensor Compensation Algorithms  
Inertial Sensor Compensation Applied To Navigation Algorithms  
Sensor Assembly Alignment Compensation

### 9. SENSOR ASSEMBLY JITTER COMPENSATION (13 Pages)

Analytical Description Of Jitter  
Jitter Rate/Acceleration Measurement  
Jitter Filter Inputs  
Jitter Removal  
Navigation Output Parameters

### 10. VIBRATION EFFECTS ANALYSIS (155 Pages)

Response To Discrete Sinusoidal Sensor Vibration Inputs  
Review Of Linear Dynamic Frequency Response Analytics  
Response To Sinusoidal System Vibration Input  
Response To Random System Vibration Input  
System Dynamic Response Analysis Model  
Vibration Effects Analysis Simulation Program

### 11. STRAPDOWN ALGORITHM VALIDATION (53 Pages)

Specialized Validation Simulations  
General Strapdown Algorithm Validation Simulators  
(Spin-Cone, Spin-Accel, Spin-Rock-Size, General Nav)

### 12. STRAPDOWN INERTIAL NAVIGATION ERROR EQUATIONS (130 Pages)

Strapdown Inertial Navigation Equations  
Navigation Error Parameters  
Navigation Error Parameter Differential Equations  
General Strapdown Inertial Sensor Error Models  
Error Equation Revisions To Enhance Quantization Noise Modeling  
Vibration Modeling

# Strapdown Analytics, Second Edition - Part 2

## Contents (805 Pages)

### 13. ANALYTICAL SOLUTIONS TO THE STRAPDOWN NAVIGATION ERROR EQUATIONS (104 Pages)

- Useful Vector Relationships
- General Navigation Error Equation Characteristics
- Navigation Errors For Constant Attitude And Constant Sensor Errors
- Navigation Errors For Rotating Attitude And Constant Sensor Errors
- Long Term Position Error For Constant Attitude And Sensor Errors
- Navigation Error From Sensor Output Random Noise During Navigation

### 14. QUASI-STATIONARY INITIALIZATION ERROR EQUATIONS AND SOLUTIONS (80 Pages)

- Fine Alignment Analytical Process Equations
- Quasi-Stationary Initial Alignment Error Equations
- Initial Alignment Errors Produced By Constant Inertial Sensor Errors
- Initial Alignment Error Caused By Ramping Accelerometer Error
- Correlation Between Sensor Errors During Initial Alignment And Navigation
- Initial Alignment Error Caused By Random Sensor Errors And Disturbances

### 15. KALMAN FILTERING TECHNIQUES (144 Pages)

- Kalman Filtering In General
- Discrete And Continuous Forms
- Suboptimal Kalman Filters
- Timing And Synchronization
- Kalman Filter Software Validation
- Examples Of Kalman Filtering Applied To Strapdown Inertial Navigation
  - (Quasi-Stationary Fine Alignment, Dynamic Moving Base INS Alignment, Using A Body Mounted Velocity Sensor, GPS - INS Position Aiding)

### 16. COVARIANCE SIMULATION PROGRAMS

- (70 Pages)
- Covariance Simulation Analytical Definition (Idealized And Delayed Control Resets)
- Suboptimal Covariance Analysis Simulation Program Configuration
  - (Program Structure, Specifying Error Models, Sensitivities And Error Budgets, Output Routines)

### 17. TRAJECTORY GENERATORS (120 Pages)

- Trajectory Shaping Function
- Segment Shaping
- Quick-Look Projection
- Adding Aerodynamic And Wind Gust Effects
- Trajectory Regeneration Function
- Segment Junction Smoothing
- Specifying Sensor Frame Orientation
- Adding Lever Arm And Vibration Effects
- Using A Trajectory Generator In Aided Strapdown INS Simulations
  - (Simulating Strapdown INS Sensors And GPS Receivers)

### 18. STRAPDOWN INERTIAL SYSTEM TESTING (132 Pages)

- Schuler Pump Test
- Strapdown Drift Test
- System Level Angular Rate Sensor Random Noise Estimation
- Strapdown Rotation Test (16 and 21 Rotation Sequence Versions)
- For Each, Analytical Theory And Detailed Test Procedures

### 19. SUPPLEMENTAL TOPICS (83 Pages)

- A Uniform Mathematical Framework For Strapdown Algorithm Design
- Analytical Modeling Of Sensor Quantization Error In Strapdown Inertial navigation Systems
- What Do Inertial Sensors Measure ?

## STRAPDOWN ANALYTICS II GENERAL ORDER FORM

**General Sales Policy:** *Strapdown Analytics, Second Edition* (Parts 1 and 2) will only be sold as a set. The Price for the set is \$185.00. Pricing is subject to change without notice.

**Ordering Information:** All orders must be prepaid by personal or company check, travelers check, international money order, or credit card (VISA, MasterCard, Diners Club or American Express). Checks must be made payable to Strapdown Associates, Inc. in U.S. dollars, drawn on a U.S. bank. Please add the appropriate shipping and handling charge for each order. We cannot accept Credit Card number/date payment information by email - Please use our telephone, Fax, normal mail, or new secure Payment Gateway (recommended) as alternates.

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